actuators and sensors would need to be appropriately located and added to the active structure. The strain-rate estimate was generated with a band-limited derivative filter, with the cutoff frequency set to 20 Hz. The filter captured the derivative estimation of the targeted first-mode bending frequency.

The slewing active structure was subjected to a near-minimum time input[11] from 0 to 40 deg in 0.317 s. Two runs were performed both with and without a payload variation. In addition, each run was performed with and without active vibration control. The reference trajectory, passive (piezo off) and active (piezo on) runs for the no-payload case are shown in Fig. 2. The slew trajectory was tracked very well with small oscillations. The root strain responses are shown in Fig. 2. With the active case, the initial residual strain was reduced by 10–15 μstrain and the settling time reduced, from greater than 10 to less than 3 s. The tip acceleration reponses are shown in Fig. 2, which demonstrate similar reductions in settling time.

The NAC algorithm was tested for robustness to parameter variations by increasing the tip mass, which reduced the first bending frequency to 4.5 Hz. The results for this run are also shown in Fig. 2. For the angle responses, the control algorithm still maintained good tracking performance with an increase in oscillations about the set point. It is believed that the piezoceramic actuator saturation does not allow the controllers to be completely decoupled. The oscillations in Fig. 2, caused by the tip mass variation, do not persist and die out after 3 s. With the tip mass variation, a similar reduction in settling time and residual vibrations are still observed for the active run. For the acceleration response, residual vibrations and settling time are reduced for time greater than 2 s; however, the increased transient spikes are due to piezoceramic actuator saturation. Reduction in these transients would require an increase in piezoceramic actuator control authority. The corresponding motor torque and piezoceramic actuator voltages for with and without a payload are shown in Fig. 2. A 50% amplitude reduction for the piezoceramic actuator was applied to prevent degradation due to large actuation voltages relative to lower vibration levels. This problem is reviewed in more detail in Ref. 12.

V. Conclusions

A robust NAC system was designed for the rotational slewing of an active structure. Experimental runs validated the control system performance. Reductions in both residual vibration and settling time resulted. Robustness to parameter variations were tested by increasing the tip mass so that the first-mode bending frequency was reduced. Control system performance results were similar to the zero tip mass case.

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Partial Eigenstructure Assignment Approach for Robust Flight Control

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Introduction

THE main purpose of a flight control system is to improve the aircraft's dynamic properties for good flying quality. For example, the well-known MIL-F-8785C (Ref. 1) gives some performance criteria for good flying quality. In this guidance, the criteria are translated into the eigenstructure specifications. Therefore, eigenstructure assignment[2,3] is particularly suitable for attaining good transient responses, and many successful applications for flight control design are presented.[4–9]

At the same time, an implemented control system should not cause instability, even though there exist parameter uncertainties, unmodeled dynamics, and so forth. However, feedback synthesis with a given robustness bound via ordinary eigenstructure assignment technique is a difficult problem, because little design freedom is left for achieving another objective such as robust stabilization.

To attain more design freedom, partial pole/eigenstructure assignment[10–16] is an effective technique. In the pioneering work,[10,11,14] pole placement of large-scale systems and eigenvalue sensitivity improvement by attained freedom are primarily considered. The dynamic properties of aircraft fundamentally consist of rigid-body dynamics and parasitic dynamics (e.g., washout filter, actuator dynamics, etc.); the former predominantly affects flying quality. Therefore, partial eigenstructure assignment is a reasonable approach. Kim and Kim[15] reduced the attained freedom to a design parameter via the null space approach and discussed an application to flight control design by linear quadratic regulator with partial eigenstructure assignment.

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In this Note, a new partial eigenstructure assignment technique via static feedback is proposed. The attained design freedom is reduced to a design parameter that is utilized for other control objectives with no disturbance of the partially assigned eigenstructures. The partial pole placement gain parameterization via polynomial matrix approach[12,13] plays an important role in the proposed technique. As an illustrative example, robust lateral-directional stability augmentation system (SAS) design using the current technique is presented.

Partial Eigenstructure Assignment

Consider a linear time-invariant system

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{1}$$

and a state feedback control law $\boldsymbol{u}(t) = -K\boldsymbol{x}(t)$, where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$, $\boldsymbol{u}(t) \in \boldsymbol{R}^m$. Assume that (A, B) is controllable and B has full column rank. For simplicity, introduce the following assumption without loss of generality. This form is easily obtained via a similar technique of the transformation to controllability canonical form.

Assumption 1:

$$A = \begin{bmatrix} A_{00} & A_{01} \\ J_1 & A_{11} \end{bmatrix}, \qquad B = \begin{bmatrix} J_0 \\ 0 \end{bmatrix}$$
$$J_1 \in \boldsymbol{R}^{(n-m)\times m}, \qquad A_{11} \in \boldsymbol{R}^{(n-m)\times(n-m)} \tag{2}$$

The parameterization of the partial pole placement gain via polynomial matrix approach[12,13] is given as

$$K = LA + \Phi L \tag{3}$$

$$L = J_0^{-1}[I_m \quad L_1] \in \boldsymbol{R}^{m\times n} \tag{4}$$

where $\Phi \in \boldsymbol{R}^{m\times m}$, $L_1 \in \boldsymbol{R}^{m\times(n-m)}$. If L (or L_1) is chosen suitably, then $n-m$ of closed-loop poles are assigned suitably. The remaining design freedom is parameterized in terms of Φ, which is independent of L.

Let us explain how L should be chosen to attain partial eigenstructure assignment. Define a self-conjugate set of scalars and a self-conjugate set of vectors with corresponding indices as $\Lambda^d := \{\lambda_i^d \in \boldsymbol{C}, i = 1,\ldots,n-m\}$, $\Xi^d := \{\boldsymbol{\xi}_i^d \in \boldsymbol{C}^n, i = 1,\ldots,n-m\}$. For a technical reason, let us introduce the next assumption with little loss of generality.

Assumption 2: $\lambda_i^d \notin \mathrm{eig}(A_{11})$.

Theorem 1: Let system (1) and a pair (Λ^d, Ξ^d) be given and Assumptions 1 and 2 hold. Define Σ_u^d and Γ_i for $i = 1,\ldots,n-m$ as

$$\Sigma_u^d := [0 \quad I_{n-m}]\left[\boldsymbol{\xi}_1^d \quad \cdots \quad \boldsymbol{\xi}_{n-m}^d\right] \tag{5}$$

$$\Gamma_i := \begin{bmatrix} -I_m \\ (A_{11} - \lambda_i^d I_{n-m})^{-1} J_1 \end{bmatrix} \tag{6}$$

Then there exists $L \in \boldsymbol{R}^{m\times n}$ such that

$$^\forall i = 1,\ldots,n-m, \qquad \left[\lambda_i^d I - (A - BK)\right]\boldsymbol{\xi}_i^d = 0 \tag{7}$$

holds for any $\Phi \in \boldsymbol{R}^{m\times m}$ where K is defined as in Eq. (3) if the following statements hold:

1) For all $i = 1,\ldots,n-m$, there exists $\boldsymbol{w}_i \in \boldsymbol{C}^m$ such that

$$\Gamma_i \boldsymbol{w}_i = \boldsymbol{\xi}_i^d \tag{8}$$

$$[\boldsymbol{w}_1 \quad \cdots \quad \boldsymbol{w}_{n-m}]\left(I_{n-m} - \Sigma_u^{d+}\Sigma_u^d\right) = 0 \tag{9}$$

2) $\boldsymbol{\xi}_i^d = \overline{\boldsymbol{\xi}_j^d}$ when $\lambda_i^d = \overline{\lambda_j^d}$.

In this case, if all $\boldsymbol{\xi}_i^d$ are real vectors, $L_1 \in \boldsymbol{R}^{m\times(n-m)}$ is given by

$$L_1 := [\boldsymbol{w}_1 \quad \cdots \quad \boldsymbol{w}_{n-m}]\Sigma_u^{d+} + \Psi\left(I_{n-m} - \Sigma_u^d\Sigma_u^{d+}\right) \tag{10}$$

for any matrix $\Psi \in \boldsymbol{R}^{m\times(n-m)}$, otherwise Σ_u^d (or $\boldsymbol{\xi}_i^d$) and $\boldsymbol{w}_i$ in Eq. (10) should be altered as

$$\left[\boldsymbol{\xi}_i^d \quad \boldsymbol{\xi}_j^d\right] := \left[\boldsymbol{\xi}_i^d \quad \boldsymbol{\xi}_j^d\right]M \tag{11}$$

$$\left[\boldsymbol{w}_i \quad \boldsymbol{w}_j\right] := [\boldsymbol{w}_i \quad \boldsymbol{w}_j]M \tag{12}$$

$$M := \frac{1}{2}\begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \tag{13}$$

when $\lambda_i^d = \overline{\lambda_j^d}$.

Proof 1: From Eqs. (3) and (4) and Assumption 1, condition (7) is rearranged as

$$\left[\lambda_i^d I_n - (A - BK)\right]\boldsymbol{\xi}_i^d = T^{-1}\begin{bmatrix} \lambda_i^d I_m + \Phi & 0 \\ J_1 J_0 & \left(\lambda_i^d I_{n-m} - A_{11}\right) + J_1 L_1 \end{bmatrix} T\boldsymbol{\xi}_i^d \tag{14}$$

where

$$T := \begin{bmatrix} J_0^{-1} & J_0^{-1}L_1 \\ 0 & I_m \end{bmatrix}, \qquad T^{-1} = \begin{bmatrix} J_0 & -L_1 \\ 0 & I_m \end{bmatrix}$$

Note that $\det(J_0) \neq 0$ from Assumption 1. Premultiplying T by (8), the following relationship holds:

$$T\boldsymbol{\xi}_i^d = \begin{bmatrix} J_0^{-1}L_1\left(A_{11} - \lambda_i^d I_{n-m}\right)^{-1}J_1 - J_0^{-1} \\ \left(A_{11} - \lambda_i^d I_{n-m}\right)^{-1}J_1 \end{bmatrix}\boldsymbol{w}_i \tag{15}$$

Therefore, from Eqs. (14) and (15), we obtain

$$\left[\lambda_i^d I_n - (A - BK)\right]\boldsymbol{\xi}_i^d = \left(\lambda_i^d I_m + J_0\Phi J_0^{-1}\right)\begin{bmatrix} L_1\left(A_{11} - \lambda_i^d I_{n-m}\right)^{-1}J_1 - I_m \\ 0 \end{bmatrix}\boldsymbol{w}_i \tag{16}$$

On the other hand,

$$\Sigma_u^d = \left[\left(A_{11} - \lambda_1^d I_{n-m}\right)^{-1}J_1\boldsymbol{w}_1 \quad \cdots \quad \left(A_{11} - \lambda_{n-m}^d I_{n-m}\right)^{-1}J_1\boldsymbol{w}_{n-m}\right] \tag{17}$$

from Eqs. (5) and (8). Hence, if there exists $L_1 \in \boldsymbol{C}^{m\times(n-m)}$ such that

$$L_1\Sigma_u^d = [\boldsymbol{w}_1 \quad \cdots \quad \boldsymbol{w}_{n-m}] \tag{18}$$

then

$$^\forall i = 1,\ldots,n-m, \qquad \left[L_1\left(A_{11} - \lambda_i^d I_{n-m}\right)^{-1}J_1 - I_m\right]\boldsymbol{w}_i = 0 \tag{19}$$

The existence condition of such L_1 is given by Eq. (9). Furthermore, all L_1 are given by Eq. (10), where $\Phi \in \boldsymbol{C}^{m\times(n-m)}$ is an arbitrary matrix. Thus, K in Eq. (3) holds condition (7) for L_1 in Eq. (10).

Let us finally prove if there exists L_1 such that Eq. (19) holds and the statement 2) holds; there always exists L_1 (or K) of the real matrix.

Assume that the column vectors of Eq. (18) with the indices (i, j) are a conjugate pair. A part of condition (18) about the conjugate pair is written as

$$L_1[0 \quad I_{n-m}]\left[\boldsymbol{\xi}_i^d \quad \boldsymbol{\xi}_j^d\right] = \left[\boldsymbol{w}_i \quad \boldsymbol{w}_j\right] \tag{20}$$

Postmultiplying the nonsingular matrix (13) by Eq. (20), we obtain the (part of the) equivalent condition by the meaning of the existence of L_1 as

$$L_1[0 \quad I_{n-m}]\left[Re\left(\boldsymbol{\xi}_i^d\right) \quad Im\left(\boldsymbol{\xi}_i^d\right)\right] = [Re(\boldsymbol{w}_i) \quad Im(\boldsymbol{w}_i)] \tag{21}$$

From Eq. (21), it is proven that there is a real-valued L_1.

Consequently, a real $L_1 \in \boldsymbol{R}^{m\times(n-m)}$ is given by Eq. (10) with a real arbitrary matrix $\Psi \in \boldsymbol{R}^{m\times m}$ if all conjugate pairs $(\boldsymbol{\xi}_i^d, \boldsymbol{\xi}_j^d)$ and $(\boldsymbol{w}_i, \boldsymbol{w}_j)$ in Eq. (18) are altered as Eqs. (11) and (12) respectively.

Note 1: There exists $\boldsymbol{w}_i$ if and only if $\boldsymbol{\xi}_i^d \in \mathrm{Img}(\Gamma_i)$ in Eq. (8). In general, the desired eigenvector $\boldsymbol{\xi}_i^d$ will not reside within $\mathrm{Img}(\Gamma_i)$; hence, eigenvector assignment is not attainable via Theorem 1. Thus, a suitable modification of the eigenvector assignment is required. The orthogonal projection of $\boldsymbol{\xi}_i^d$ over $\mathrm{Img}(\Gamma_i)$ is the best alteration. It is given as $\hat{\boldsymbol{\xi}}_i^d := \Gamma_i(\Gamma_i^*\Gamma_i)^{-1}\Gamma_i^*\boldsymbol{\xi}_i^d$, which minimizes $\|\boldsymbol{\xi}_i^d - \hat{\boldsymbol{\xi}}_i^d\|_2$. This idea is familiar in the ordinary (total) eigenstructure assignment technique.[3]

Note 2: From Eq. (8), the allowable eigenvector subspace corresponding to λ_i [i.e., $\mathrm{Img}(\Gamma_i)$] is spanned by the column vectors of Γ_i in Eq. (6) and the dimension of the subspace is m. This constraint is the same as the ordinary (total) eigenstructure assignment approach.[3,5,6]

For simplicity, let us introduce the next assumption.

Assumption 3: $\boldsymbol{\xi}_i^d \in \mathrm{Img}(\Gamma_i)$ and $\det(\Sigma_u^d) \neq 0$.

In many practical situations, it is allowable to modify the eigenvector specifications to satisfy Assumption 3. Consequently, a procedure for choosing a suitable L is summarized as follows.

Procedure 1: Let system (1) and specifications on $n-m$ closed-loop eigenstructure be given and Assumptions 1, 2, and 3 hold. Step 1: Compute Σ_u^d in Eq. (5) and Γ_i in Eq. (6) for $i = 1, \ldots, n-m$. Step 2: Obtain $\boldsymbol{w}_i$ by solving linear equation (8) for each i. From Assumption 3, Eq. (9) holds. Step 3: Choose $L_1 := [\boldsymbol{w}_1 \ \cdots \ \boldsymbol{w}_{n-m}](\Sigma_u^d)^{-1}$. It is ensured that $\det(\Sigma_u^d) \neq 0$ from Assumption 3. Step 4: The suitable L is given as in Eq. (4) for the L_1 in step 3. It is proved via Theorem 1 that Eq. (7) holds for K in Eq. (3) and arbitrary $\Phi \in \boldsymbol{R}^{m\times m}$.

Robust Stabilization with Partial Eigenstructure Assignment

We show that the proposed eigenstructure assignment technique is easily applicable to the modern robust control technique. This significant feature is due to the two characteristic eigenstructure sets corresponding to the design parameters, L and Φ, respectively. Designers can assign $n-m$ eigenstructures by choosing L for suitable time-domain performance and mode decoupling.

The design freedom of the remaining closed-loop eigenstructure is unified as Φ. Designers can utilize Φ for robust stabilization of the entire feedback system.

Let us consider an extended system with output $\tilde{\boldsymbol{y}} = L\boldsymbol{x}$. Then Φ is considered a fictitious *output* feedback gain. The assigned eigenstructures are not disturbed by any Φ (see Refs. 12 and 13), because the assigned modes of the extended system are not observable. Consequently, designers can apply various modern robust control techniques such as quadratic stabilization, H_∞ control, and μ design via (fictitious) output feedback of Φ.

Let us assume that a suitable L is already given from eigenstructure assignment specifications. Therefore, we describe how to choose suitable Φ for attaining quadratic stability. Consider a linear system with structured time-variant perturbation,

$$\dot{\boldsymbol{x}}(t) = [A + \Delta A(t)]\boldsymbol{x}(t) + [B + \Delta B(t)]\boldsymbol{u}(t) \tag{22}$$

$$[\Delta A(t) \quad \Delta B(t)] := D\Delta(t)[E_a \quad E_b], \quad {}^\forall t \geq 0, \quad \|\Delta(t)\| \leq 1 \tag{23}$$

and assume that (A, B) is controllable, (A, D) is stabilizable, B has full-column rank, and $\Delta(t)$ is Lebesgue measurable. Note that the nominal system [i.e., $\Delta(t) \equiv 0$] of Eq. (22) is system (1).

It is proved that the following two problems are equivalent in consideration of a well-known relationship between quadratic stabilization and H_∞ control.[17]

Problem 1: Consider the uncertain system (22). Given L, find Φ such that the system is robustly stabilized against the structured uncertainty (23) by a state feedback gain (3).

Problem 2: Consider a generalized plant

$$\dot{\boldsymbol{x}}(t) = (I - BL)A\boldsymbol{x}(t) + D\boldsymbol{w} + B\tilde{\boldsymbol{u}}(t) \tag{24}$$

$$\boldsymbol{z}(t) = (E_a - E_bLA)\boldsymbol{x}(t) + E_b\tilde{\boldsymbol{u}}(t) \tag{25}$$

$$\tilde{\boldsymbol{y}}(t) = L\boldsymbol{x}(t) \tag{26}$$

which is decided by Eq. (22) and a given L. Find an output feedback law $\tilde{\boldsymbol{u}}(t) = -\Phi\tilde{\boldsymbol{y}}(t)$ such that a closed-loop system is internally stable and $\|T_{zw}\|_\infty < 1$, where T_{zw} means a transfer function from $\boldsymbol{w}$ to $\boldsymbol{z}$.

Therefore, we can find suitable Φ via the equivalent H_∞ control problem, Problem 2.

Theorem 2: Consider Problem 1. There exists Φ which is a solution of Problem 1 if and only if there exist symmetric matrices $X \in \boldsymbol{R}^{n\times n}$ and $Y \in \boldsymbol{R}^{n\times n}$ such that

$$X > 0 \tag{27}$$

$$\hat{B}^\perp \begin{bmatrix} \hat{A}Y + Y\hat{A}^T & D & Y\hat{E}_a^T \\ D^T & -I & 0 \\ \hat{E}_aY & 0 & -I \end{bmatrix} (\hat{B}^\perp)^T < 0 \tag{28}$$

$$\hat{D}^\perp \begin{bmatrix} X\hat{A} + \hat{A}^TX & XD & \hat{E}_a^T \\ D^TX & -I & 0 \\ \hat{E}_a & 0 & -I \end{bmatrix} (\hat{D}^\perp)^T < 0 \tag{29}$$

$$X = Y^{-1} \tag{30}$$

where $\hat{A} := (I - BL)A$, $\hat{B} := [B^T \ 0 \ E_b^T]^T$, $\hat{E}_a := E_a - E_bLA$, $\hat{D} := [-L \ 0 \ 0]^T$. Furthermore, Φ is given by a solution of linear matrix inequality (LMI),

$$\hat{Q}(Y) + \hat{B}\Phi\hat{C}(Y) + \hat{C}(Y)^T\Phi^T\hat{B}^T < 0 \tag{31}$$

where Y is such that Eqs. (27–30) hold and

$$\hat{Q}(Y) := \begin{bmatrix} \hat{A}Y + Y\hat{A}^T & D & Y\hat{E}_a^T \\ D^T & -I & 0 \\ \hat{E}_aY & 0 & -I \end{bmatrix}$$

$$\hat{C}(Y) := [-LY \quad 0 \quad 0]$$

Proof 2: Considering the equivalence between Problems 1 and 2,[18,19] it is proven that system (22) is robustly stabilized by the state feedback gain (3) if and only if there exists Φ such that $\|T_{zw}\|_\infty < 1$ and the closed-loop system is internally stabilized. From the bounded real lemma, such Φ exists if and only if there exists $Y > 0$ such that Eq. (31) holds. From Theorem 4.2 in Ref. 20, there exists Φ such that Eq. (31) holds for a given $Y > 0$ if and only if Eq. (28) and the next condition hold:

$$[\hat{C}(Y)^T]^\perp \begin{bmatrix} \hat{A}Y + Y\hat{A}^T & D & Y\hat{E}_a^T \\ D^T & -I_q & 0 \\ \hat{E}_aY & 0 & -I_q \end{bmatrix} \{[\hat{C}(Y)^T]^\perp\}^T < 0 \tag{32}$$

From the next relationship,

$$\hat{F}(Y)^\perp = \hat{D}^\perp S, \; S := \begin{bmatrix} Y^{-1} & 0 & 0 \\ 0 & I_q & 0 \\ 0 & 0 & I_q \end{bmatrix}$$

condition (32) is equivalent to Eqs. (29) and (30). Thus, if there exists (X, Y) such that Eqs. (28–30) hold, then there exists Φ such that Eq. (31) holds. Conversely, if there exists Φ such that Eq. (31) holds for some Y, then conditions (28–30) hold for Y.

Consequently, there exists Φ such that Eq. (31) holds if and only if there exists (X, Y) such that Eqs. (28–30) hold. For such Y and Φ, Eq. (31) obviously holds. Furthermore, system (22) is robustly stabilized by the state feedback gain (3) for the given L and Φ.

Problem 2 is a kind of low-order H_∞ control problem, which generally is a nonconvex optimization problem. A practical effective numerical optimization algorithm is proposed in the literature.[21,22] Now let us summarize our design procedure.

Procedure 2: Let a system with structured uncertainty system (22), (23) and L be given. Assume that L has been chosen suitably via Procedure 1. Step 1: Find (X, Y) that satisfy Eqs. (27–30) via numerical optimization approach (e.g., the algorithms in Refs. 21 and 22). Step 2: Obtain Φ by solving the LMI (31), which can be easily solved by using MATLAB LMI Toolbox. Determine the state feedback gain (3). It is proven via Theorems 1 and 2 that quadratic stabilization and assignment of partial eigenstructure are simultaneously achieved by this K.

Illustrative Example

To illustrate our design method, we present a simple lateral/directional SAS design.[23] Consider fifth-order linearized dynamics of a lightweight aircraft, which is described as

$$\dot{\boldsymbol{x}}(t) = [A + \Delta A(t)]\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{33}$$

$$A = \begin{bmatrix} -0.228 & 0.059 & -0.998 & -0.0037 & 0.0395 \\ -34.9 & -1.516 & 0.872 & 21.3 & 9.92 \\ 18.69 & 0.0379 & -0.564 & 0.504 & -8.29 \\ 0 & 0 & 0 & -40 & 0 \\ 0 & 0 & 0 & 0 & -25 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 40 & 0 \\ 0 & 0 & 0 & 0 & 25 \end{bmatrix}^T \tag{34}$$

$$\boldsymbol{x} = \begin{bmatrix} \beta \\ p \\ r \\ \delta_a \\ \delta_r \end{bmatrix}, \qquad \boldsymbol{u} = \begin{bmatrix} \delta_{\mathrm{ac}} \\ \delta_{\mathrm{rc}} \end{bmatrix} \tag{35}$$

where

p = roll rate, rad/s
r = yaw rate, rad/s
β = sideslip angle, rad
δ_a = aileron deflection angle, rad
δ_{ac} = aileron command, rad
δ_r = rudder deflection angle, rad
δ_{rc} = rudder command, rad

and where the aileron and the rudder actuator dynamics are included in Eq. (33) as first-order lags with the time constant $1/40$ and $1/25$ s, respectively.

Assume that system (33) has individual perturbations of $\pm 40\%$ in $A(2,1)$, $A(2,4)$, $A(3,1)$, and $A(3,5)$. Hence, D, E_a, E_b, and $\Delta(t)$ in Eq. (23) are given as follows:

$$D = 0.4 \begin{bmatrix} 0 & 0 & 0 & 0 \\ A(2,1) & 0 & A(2,4) & 0 \\ 0 & A(3,1) & 0 & A(3,5) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad E_b = [0_{4\times 2}]$$

$\Delta(t) = \mathrm{diag}\{\Delta_1(t), \Delta_2(t), \Delta_3(t), \Delta_4(t)\}$, $\forall t \geq 0$, $\|\Delta_i(t)\| < 1$, $i = 1, \ldots, 4$. In this example, actuator modeling error and washout filter are omitted for simplicity and a clear illustration of the presented idea.

The aircraft model has two rigid-body modes, roll and Dutch roll. Eigenstructure of these modes has a great influence on flying qualities of the aircraft; therefore, their eigenvalues and corresponding eigenvectors should be suitably assigned to satisfy design requirements and performance criteria. In this case, three ($n - m = 3$) poles are assignable. According to MIL-F-8785C (Ref. 1) specification and the mode decoupling requirement, we choose the desired eigenstructure $(\lambda_i^d, \boldsymbol{\xi}_i^d)$ as follows. Roll mode is (cf. open-loop pole: -1.3201) $\lambda_R^d = -3.00$, $\boldsymbol{\xi}_R^d = [0\ 1\ 0\ *\ *]^T$. Dutch roll mode is (cf. open-loop poles: $-0.4939 \pm 4.5249j$) $\lambda_D^d = -1.50 \pm 3.50j$, $\boldsymbol{\xi}_D^d = [1\ 0\ *\ *\ *]^T \pm j[1\ 0\ *\ *\ *]^T$, where '$*$' elements mean that they are free. Note that no specification is assigned to the remaining modes (i.e., actuator internal modes).

In many practical situations, we are interested in specifying partial elements of an eigenvector. It can be made easy by choosing the desired eigenvector $\boldsymbol{\xi}_i^d$ as an arbitrary solution of the linear equality $F_i \boldsymbol{\xi}_i^d = \boldsymbol{u}_i^d$. For example, in this case, F_i and $\boldsymbol{u}_i^d$ should be

$$F_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{u}_R^d = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$F_D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{u}_D^d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and a suitable $\boldsymbol{\xi}_i^d$ can be taken as $F_i^T (F_i F_i^T)^{-1} \boldsymbol{u}_i^d$.

According to Procedure 1, suitable L for roll and Dutch roll mode assignments is chosen as

$$L = \begin{bmatrix} -0.0336 & 0.0014 & 0.0040 & 0.0250 & 0 \\ -0.0254 & 0.0012 & -0.0104 & 0 & 0.0400 \end{bmatrix} \tag{36}$$

To compute the (X, Y) at step 2 in Procedure 2, the linearization algorithm proposed by Ghaoui et al.[21] is used. Then Φ, which is a

Table 1 Design results: Eigenstructures

		Mode vectors				
Mode	Poles	β	p	r	δ_a	δ_r
Roll	−3.0000	−0.0186	1.0000	0.0064	−0.0816	−0.0405
	(−3.00)	(0)	(1)	(0)	(*)	(*)
Dutch roll						
Real part	−1.5000	1.0000	0.0000	10.9098	−0.4199	3.4608
	(−1.50)	(1)	(0)	(*)	(*)	(*)
Imag. part	3.5000	1.0000	0.0000	0	1.3425	0.6356
	(3.50)	(1)	(0)	(*)	(*)	(*)
Rudder	−10.430	−0.0308	1.0000	−0.2699	−0.2704	−0.4025
Aileron	−50.813	0.0029	−0.4136	0.1593	0.4896	1.0000

solution of the LMI (31), is obtained as follows:

$$\Phi = \begin{bmatrix} -72.8338 & 39.9740 \\ -257.5513 & 134.0772 \end{bmatrix} \tag{37}$$

Thus, a suitable feedback gain is designed as

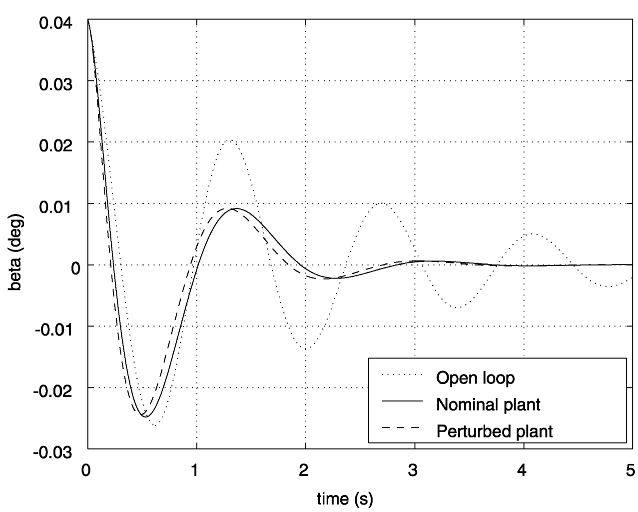
$$K = \begin{bmatrix} 1.4627 & -0.0567 & -0.6758 & -2.7891 & 1.5779 \\ 5.0047 & -0.1990 & -2.3966 & -6.4181 & 4.4600 \end{bmatrix} \tag{38}$$

via Procedure 2.

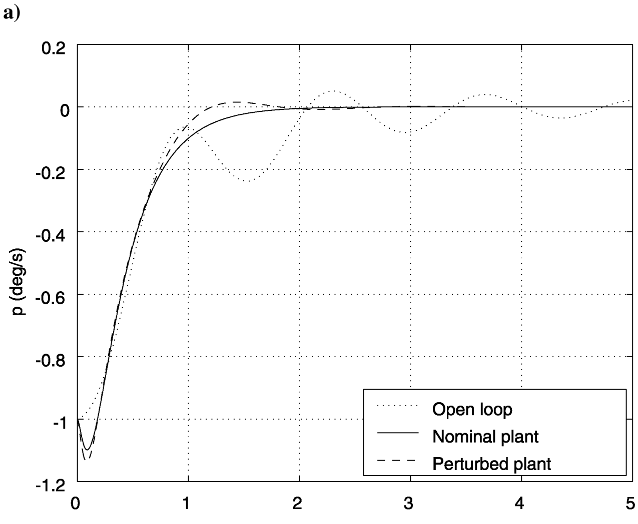
The eigenstructure assignment results are shown in Table 1. The value in parentheses indicates the design specifications. Recall that no excessive specifications are given for the remaining modes (rudder and aileron actuator internal modes). The exact assignment of the roll and Dutch roll poles are due to the parameterization of the partial pole assignment gain.

In this result, $\|T_{zw}\|_\infty = 0.940(<1)$; hence quadratic stability is attained and system (33) is robustly stabilized against the specified structured uncertainty.

The performance results are shown in Fig. 1. The labels "nominal plant" and "perturbed plant" represent the $\Delta(t) \equiv 0$ case and the $\Delta(t) \equiv I_4$ case, respectively. The response of the system for the initial state $\boldsymbol{x}(0) = [0.04 \text{ (deg)}, -1 \text{ (deg/s)}, 0, 0, 0]^T$ is shown in Fig. 1. Dutch roll mode is well damped and mode decoupling is fully improved as the result of rigid-body mode assignment. Although the assigned eigenstructures are also perturbed, it is observed that specified performance is fully achieved in the perturbed plant cases.



a)



b)

Fig. 1 Initial responses: initial state $x(0)$ = [0.04 (deg), −1 (deg/s), 0, 0, 0]^T.

Conclusions

It is well known that eigenstructure assignment is a powerful technique for transient response shaping. In this Note, we proposed a new partial eigenstructure assignment technique that is easily applicable to the modern robust control technique. In this approach, eigenstructure specifications are only given for partial modes; therefore, more freedom is acquired for the feedback design than the ordinary (total) eigenstructure assignment approach. Partial eigenstructure assignment is achieved easily by choosing suitable L, and other control objectives are ensured by suitably choosing another design parameter Φ. The partial eigenstructure which is assigned by L is not disturbed by any Φ.

To demonstrate the effectiveness of the presented technique, a mode decoupling robust flight control design by the state feedback, which simultaneously achieves quadratic stabilization and partial eigenstructure assignment, is presented.

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Linear System Input-Order Reduction by Hankel Norm Maximization

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I. Introduction

CONTROL blending and disturbance direction identification are two examples of linear system input-dimension reduction problems. In control blending problems, system input dimension is reduced to facilitate control design and implementation. Typically, the system has dynamically redundant actuators that are to be mapped to a smaller set of generalized controls and an associated control mapping.

In disturbance decoupling problems where the disturbance is meant to model neglected higher-order or nonlinear dynamics, determination of the direction from first principles is not always practical or possible. When the disturbance direction is found empirically, typically several directions are found, each one associated with a different operating point, and a suitable representative direction must be chosen.[1] The problem is complicated further when the rank of the disturbance map is not known, that is, when it is not clear how many directions should be chosen from the empirically derived set.

One approach to model input reduction, appealing at first, is to inspect, pairwise, the angle between each input direction. If all the angles are small it might seem reasonable to choose any one direction or an average of all of them as a rank-one direction representative of the set.[1] However, given that the input directions, form a linearly independent set, the angles depend on the chosen state-space basis. For $m \leq n$ linearly independent input directions, where n is the dimension of the state space, state transformations may be found that make the pairwise angles between the directions arbitrarily close to 0 or 90 deg. It is, therefore, not clear whether the rank of the input map should be 1 or m or something in between.

A second approach is to group the input directions into a single multi-input mapping and to consider the singular values of this map.[2,3] If B_i is a linear system input map and $B = [B_1, \ldots, B_m]$ is an aggregate mapping, a reduced rank map is formed by combining the left singular vectors associated with the largest singular values of B, that is, once a threshold is chosen for what is to be considered a small singular value. This approach amounts to the first because the left singular vectors and singular values of B depend on the basis chosen for the state space unless, of course, only unitary transformations on the state space are allowed.

An underlying difficulty with both approaches is that the dynamics and input–output characteristics of the system are neglected. Consider, for example, a disturbance direction identification problem. A worst-case disturbance often is measured in an ℓ_2 sense, which suggests an $\mathcal{H}_\infty$ norm as the important system property to be maximized when performing a model approximation. On the other hand, in a control-blending problem, controllability and observability are the important system properties. A Hankel norm provides such an indicator as the square root of the largest eigenvalue of the product of controllability and observability gramians. Thus, the choice of the cost used in performing the model input reduction depends on the role the input plays in the system dynamics.

The following section introduces some notation. Section III presents an approach to model input reduction based on maximizing a system $\mathcal{H}_\infty$ norm. Section IV presents an approach based on maximizing a system Hankel norm. Section V illustrates an application to a disturbance direction identification problem; a numerical example is included. Section VI illustrates an application to a control-blending problem; a numerical example is also included.

II. Notation

Let G be a linear time-invariant system with k multidimensional inputs:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B_1\boldsymbol{u}_1 + \cdots + B_k\boldsymbol{u}_k, \qquad \boldsymbol{y} = C\boldsymbol{x}$$

The system has n states and p outputs and the k inputs $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_k$ each have dimension m_i. It is also useful to define an aggregate-input mapping $B = [B_1, \ldots, B_k]$ that operates on an m-dimensional vector $\boldsymbol{u}$ where $m = \sum m_i$. Thus, if the inputs belong to real vector spaces as in $\boldsymbol{u}_i \in \mathcal{U}_i$ and $\boldsymbol{u} \in \mathcal{U}$, then the $\mathcal{U}_i$ form an orthogonal decomposition of $\mathcal{U}$ as in $\mathcal{U} = \mathcal{U}_1 \oplus \cdots \oplus \mathcal{U}_k$. A single-input system derived from G is indicated by G_q and defined as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{q}\bar{u}, \qquad \boldsymbol{y} = C\boldsymbol{x}$$

where $\bar{u}$ is a real scalar, $\boldsymbol{q} : \mathcal{R} \mapsto \mathcal{R}^m$ and $\|\boldsymbol{q}\| = 1$. Finally, L_c and L_{cq} are the controllability gramians of the pairs (A, B) and $(A, B\boldsymbol{q})$. L_o is the observability gramian of the pair (C, A).

III. $\mathcal{H}_\infty$ Norm Maximization Approach

In a disturbance modeling problem, the worst-case disturbance direction, in an ℓ_2 sense, is indicated by maximizing the system $\mathcal{H}_\infty$ norm. The $\mathcal{H}_\infty$ norm of the full-order system (C, A, B) may be regarded as a search over frequency for the largest singular value of the frequency response matrix:

$$\|G\|_\infty = \max_\omega \left\{\sigma_{\max}[C(\jmath\omega I - A)^{-1}B]\right\} \tag{1}$$

The search may be tuned by applying frequency-dependent weights to the input and output. Form a model input reduction problem by finding q such that the $\mathcal{H}_\infty$ norm of the system with reduced-order

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